

Introduction to Logic (CS & MA)

Answers midterm exam

1. (a) $(\neg B \wedge C) \rightarrow \neg A$
A: The weather is bad.
B: It rains.
C: The sun shines.
- (b) $A \rightarrow (B \wedge C)$
A: The human body has enough oxygen.
B: The heart pumps blood around.
C: The lungs work.
2. (a) $\neg V(g, h) \rightarrow (V(a, g) \wedge V(e, g))$ (or: $(V(a, g) \wedge V(e, g)) \vee V(g, h)$)
a: Amir
e: Ethel
g: Greg
h: Hans
V(x, y): *x* votes for *y*.
- (b) $B(b, a) \rightarrow (H(j) \wedge \neg H(k))$
j: Jaime
k: Karin
b: Barcelona
a: Ajax
H(x): *x* is happy
B(x, y): *x* beats *y*.

3. (a)	$1. \neg P \rightarrow \perp$ $2. \neg P$ $3. \perp$ $4. \neg\neg P$ $5. P$	\rightarrow Elim: 2, 1 \neg Intro: 2-3 \neg Elim: 4
(b)	$1. (P \wedge Q \wedge R) \rightarrow S$ $2. P$ $3. Q$ $4. R$ $5. P \wedge Q \wedge R$ $6. S$ $7. R \rightarrow S$ $8. Q \rightarrow (R \rightarrow S)$ $9. P \rightarrow (Q \rightarrow (R \rightarrow S))$	\wedge Intro: 2, 3, 4 \rightarrow Elim: 5, 1 \rightarrow Intro: 4-6 \rightarrow Intro: 3-7 \rightarrow Intro 2-9
(c)	$1. \neg P(a) \wedge (a = b \vee P(b))$ $2. a = b$ $3. \neg P(a)$ $4. \neg P(b)$ $5. \neg P(b)$ $6. a = b \vee P(b)$ $7. a = b$ $8. a = b$ $9. P(b)$ $10. \perp$ $11. a = b$ $12. a = b$ $13. a = b \leftrightarrow \neg P(b)$	\wedge Elim: 1 $=$ Elim: 3, 2 \wedge Elim: 1 Reit: 7 \perp Intro: 9, 5 \perp Elim: 10 \vee Elim: 6, 7-8, 9-11 \leftrightarrow Intro: 2-5, 6-12

4. (a)

P	Q	R	$((P \vee Q) \vee R) \wedge \neg Q$	$\wedge (Q \leftrightarrow R)$				
T	T	T	T	T				
T	T	F	T	F				
T	F	T	T	F				
T	F	F	T	T				
F	T	T	T	T				
F	T	F	T	F				
F	F	T	F	F				
F	F	F	F	T				
1	2	3	4	5	7	6	9	8

The numbers in the last row indicate the order in which the columns are computed. The final column (numbered 9) does not contain only the value F, so the sentence is *not a contradiction*.

(b) We abbreviate $a = a$ to I , $\text{Tet}(a)$ to T , and $\text{Small}(b)$ to S .

I	T	S	$((I \vee T) \wedge \neg T) \vee \neg(S \rightarrow \neg I)$	$\Leftrightarrow (I \wedge T) \rightarrow S$								
T	T	T	T	T								
T	T	F	T	F								
T	F	T	T	T								
T	F	F	T	T								
*	F	T	T	T								
*	F	F	T	T								
*	F	T	F	T								
*	F	F	F	T								
1	2	3	4	6	5	10	9	8	7	13	11	12

In the rows beginning with *, I (the abbreviation of $a = a$) has truth value F, which is logically impossible. So these are *spurious rows*.

The numbers in the last row indicate the order in which the columns are computed. The final column (numbered 13) only contains the value T in the non-spurious rows, so the two sentences are *logically equivalent*.

$$\begin{aligned}
5. \quad (a) \quad & \neg(\neg\neg P \wedge (Q \vee \neg R)) \\
& \Leftrightarrow \\
& \neg\neg\neg P \vee \neg(Q \vee \neg R) \\
& \Leftrightarrow \\
& \neg\neg\neg P \vee (\neg Q \wedge \neg\neg R) \\
& \Leftrightarrow \\
& \neg P \vee (\neg Q \wedge R)
\end{aligned}$$

$$\begin{aligned}
(b) \quad & \neg(P \leftrightarrow (Q \rightarrow R)) \\
& \Leftrightarrow \\
& \neg((P \rightarrow (Q \rightarrow R)) \wedge ((Q \rightarrow R) \rightarrow P)) \\
& \Leftrightarrow \\
& \neg((\neg P \vee \neg Q \vee R) \wedge (\neg(\neg Q \vee R) \vee P)) \\
& \Leftrightarrow \\
& \neg(\neg P \vee \neg Q \vee R) \vee \neg(\neg(\neg Q \vee R) \vee P) \\
& \Leftrightarrow \\
& (\neg\neg P \wedge \neg\neg Q \wedge \neg R) \vee (\neg\neg(\neg Q \vee R) \wedge \neg P) \\
& \Leftrightarrow \\
& (P \wedge Q \wedge \neg R) \vee ((\neg Q \vee R) \wedge \neg P) \\
& \Leftrightarrow \\
& (P \wedge Q \wedge \neg R) \vee (\neg Q \wedge \neg P) \vee (R \wedge \neg P)
\end{aligned}$$

6.

- | | |
|------------|------------|
| (a) False. | (f) True. |
| (b) False. | (g) False. |
| (c) False. | (h) False. |
| (d) True. | (i) True. |
| (e) False. | (j) True. |

7.	$1. (P \rightarrow Q) \rightarrow P$	
	$2. \neg P$	
	$3. P$	
	$4. \perp$	\perp Intro: 3, 2
	$5. Q$	\perp Elim: 4
	$6. P \rightarrow Q$	\rightarrow Intro: 3–5
	$7. P$	\rightarrow Elim: 6, 1
	$8. \perp$	\perp Intro 7, 2
	$9. \neg\neg P$	\neg Intro: 2–8
	$10. P$	\neg Elim: 9
	$11. ((P \rightarrow Q) \rightarrow P) \rightarrow P$	\rightarrow Intro: 1–10

Alternative proof:

1.	$\neg(((P \rightarrow Q) \rightarrow P) \rightarrow P)$	
	$2. (P \rightarrow Q) \rightarrow P$	
	$3. P$	
	$4. (P \rightarrow Q) \rightarrow P$	Reit: 3
	$5. P$	\rightarrow Intro: 4–5
	$6. ((P \rightarrow Q) \rightarrow P) \rightarrow P$	\perp Intro: 6, 1
	$7. \perp$	\perp Elim: 7
	$8. Q$	\rightarrow Intro: 3–8
	$9. P \rightarrow Q$	\rightarrow Elim: 9, 2
	$10. P$	\rightarrow Intro: 2–10
	$11. ((P \rightarrow Q) \rightarrow P) \rightarrow P$	\perp Intro: 11, 1
	$12. \perp$	\neg Intro: 1–12
	$13. \neg\neg(((P \rightarrow Q) \rightarrow P) \rightarrow P)$	\neg Elim: 13
	$14. ((P \rightarrow Q) \rightarrow P) \rightarrow P$	